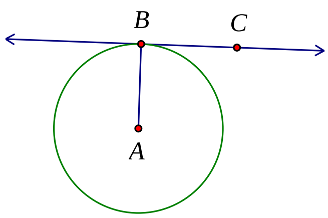
AB > AB and AB = AB is a contradiction	Definition of contradiction

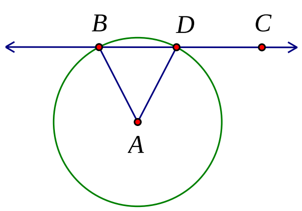
Since AB > AB is a contradiction, we conclude that our assumption is false. Therefore, $\overleftrightarrow{BC} \perp \overline{AB}$.

Perpendicular line to a radius theorem: A line that is perpendicular to a radius at the outer endpoint is tangent to the circle.



Let $\overleftrightarrow{BC}$ be a line perpendicular to radius $\overline{AB}$ of circle C(A, AB) at point B. We want to prove that $\overleftrightarrow{BC}$ is tangent to C(A, AB).

Proof (by contradiction)



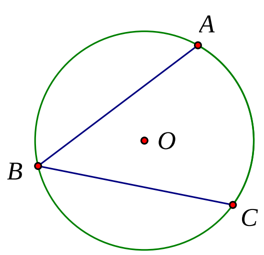
Assume that line $\overleftrightarrow{BC}$ is not tangent to C(A, AB). That is, $\overleftrightarrow{BC}$ is secant to C(A, AB).	Method of Contradiction Assumption
Line $\overleftrightarrow{BC}$ intersects C(A, AB) at two points, say B and D.	Definition of secant line
∠ABD is a right angle	Definition of perpendicular lines (segments)
m(∠ABD) = 90°	Definition of a right angle
$\overline{AB}$ and $\overline{AD}$ are radii	Definition of radii
$\overline{AB} \cong \overline{AD}$	All radii of a circle are congruent
ΔABD is isosceles	Definition of isosceles triangle
∠ABD ≅ ∠ADB	Isosceles triangle theorem
m(∠ABD) = m(∠ADB)	Definition of congruent angle

$m(\angle ADB) = m(\angle ABD)$	Symmetry property of equality
$m(\angle ADB) = 90°$	Transitive or substitution property of equality.
∠ADB is a right angle	Definition of right angle
ΔABD has two right angles	
This contradicts the fact that a triangle cannot have more than 1 right angle. Therefore, our assumption is false. That is, $\overleftrightarrow{BC}$ is tangent to C(A, AB).	

The inscribed angle theorem. The measure of an inscribed angle in a circle is half the measure of its intercepted arc.

Proof

Let ∠ABC be an inscribed angle in a circle.
We want to show that $m(\angle CDE) = m(CE)/2$

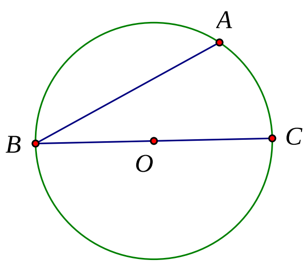


We are proving this theorem by cases:

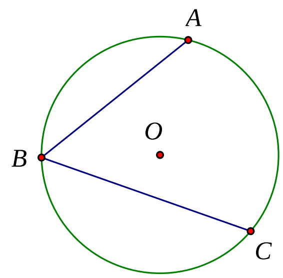
Case 1) One of the sides of the angle contains the center of the circle.

Case 2) The center of the circle is in the interior of the angle.

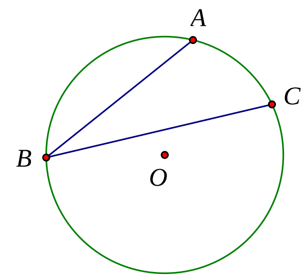
Case 3) The center of the circle is in the exterior of the angle.



Case 1

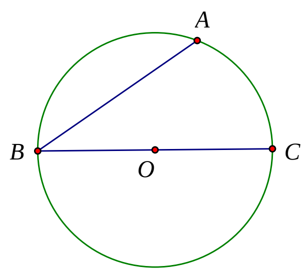


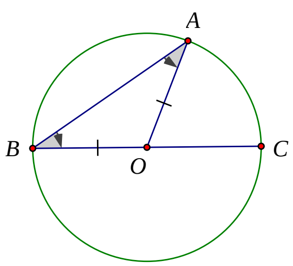
Case 2



Case 3

Case 1) One of the sides of the angle contains the center of the circle.



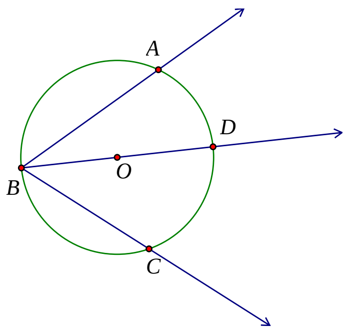


Let ∠ABC be an inscribed angle in a circle. We want to show that $m(\angle ABC) = \frac{m(\widehat{AC})}{2}$

Construct radius $\overline{OA}$	Line axiom
$\overline{OB} \cong \overline{OA}$	All radii of a circle are congruent
ΔABO is isosceles	Definition of isosceles triangle
∠ABO ≅ ∠BAO	Isosceles triangle theorem
$m(\angle ABO) = m(\angle BAO)$	Definition of congruent angles
$m(\angle AOC) = m(\angle ABO) + m(\angle BAO)$	Exterior angle theorem

$m(\angle AOC) = m(\angle ABO) + m(\angle ABO)$	Substitution property of equality
$= 2\, m(\angle ABO)$	
$m(\angle ABO) = \frac{m(\angle AOC)}{2}$	
$m(\angle AOC) = m(\widehat{AC})$	Central angle axiom
$m(\angle ABO) = \frac{m(\widehat{AC})}{2}$	Substitution property of equality

Case 2) The center of the circle is in the interior of the angle.



Let $\angle ABC$ be an inscribed angle whose interior contains the center of the circle. We want to prove that $m(\angle ABC) = \frac{m(\widehat{AC})}{2}$.

Construct $\overrightarrow{BO}$	Line axiom
$m(\angle ABD) = \frac{m(\widehat{AD})}{2}$	Case 1 of inscribed angle theorem
$m(\angle DBC) = \frac{m(\widehat{DC})}{2}$	Case 1 of inscribed angle theorem
$m(\angle ABC) = m(\angle ABD) + m(\angle DBC)$	Addition angle theorem
$= \frac{m(\widehat{AD})}{2} + \frac{m(\widehat{DC})}{2}$	Substitution property of equality
$= \frac{m(\widehat{AD})+m(\widehat{DC})}{2}$	
$= \frac{m(\widehat{AC})}{2}$	