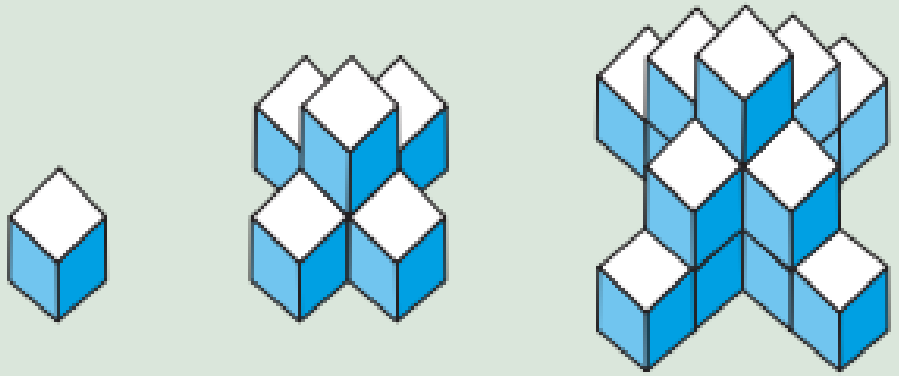

Tower 1 Tower 2 Tower 3

Use the blocks to build the fourth tower in the sequence. How many cubes did you use? How many cubes would you need to build the fifth tower? The 12th tower? The 20th tower? The 100th tower? Write a rule to help you find the number of cubes for the *n*th tower.

Adapted from Stoker (2006)

Each of the cards has a snippet of dialog between a teacher and student(s). You will place each card on the game-board in the numbered spot that you think represents the level of mathematical dialog. 0 being the lowest, and 3 being the highest. Once all of the cards are placed, try to figure out the characteristic for each level.

Level 0	Level 1
Level 2	Level 3

S: I don’t think my tower looks right.
T: No, it doesn’t appear to be correct. Maybe you should try again.

S: I don’t think my tower looks right.
T: Hmm, you’re right. Maybe James can help you sort it out.

S: I don’t think my tower looks right.
T: What makes you think that?
S: It doesn’t look like the picture.
T: James, what do you think? Is this right?

S: I don’t think my tower looks right.
T: Why do you think that?
S: It doesn't look like the picture.
T: Where do you think you went wrong?
S: I think I’ve got too many blocks on one side.
T: Maybe you can try fixing it. If you need help, someone who’s tower looks right may be able to help.

T: So the first level has 1 block, the second 5, the third 9. It's going up by 4 each time. How many blocks will be on the fourth level? S: Thirteen? T: 9 plus 4 is thirteen. So there will be thirteen blocks on the fourth level.	T: How many blocks are on the first level? S: One T: How many on the second level? S: Five T: How many on the third level? S: Nine T: So how many would be on the fourth level? S: Thirteen? T: Right.
T: Do you see a pattern in the number of blocks for each level? S: There seems to be four more blocks for each level. T: James, do you agree? J: Yes, I think there will be for more blocks on each level.	T: Do you see a pattern in the number of blocks for each level? S: There seems to be four more blocks for each level. T: Why? S: Well, there's one on the first, five on the second, and nine on the third. T: James, do you agree? J: Yes, I think there will be for more blocks on each level.

T: So we see a pattern of adding four blocks with each level.
T: The rule is then 2n(n-1)+n.
T: How many blocks would be on the sixth level?
S: 17
T: Write down the number of blocks in the 12th, 20th, and 100th tower.

T: So we see a pattern of adding four blocks with each level.
T: The rule is then 2n(n-1)+n.
T: Do you agree?
S: Yes
T: Write down the number of blocks in the 12th, 20th, and 100th tower.

T: Please explain how you found the rule for the towers.
S. The center of each tower has the same number of cubes as the tower number, so that equals n cubes.
T: Okay, then what?
S. There are four arms coming out from the center in the shape of triangles.
T: Triangles?
S. Yeah, when you flip them over you get two rectangles. The height of the rectangle is the same as the center, and the width is one less. So $2n(n - 1) + n$ gives you the number of cubes.
T: $2n(n - 1) + n$. Does everyone agree? Does everyone understand how he got the answer? Okay, who else has a solution?

S. The center of each tower has the same number of cubes as the tower number, so that equals n cubes.
T: Okay, then what?
S. There are four arms coming out from the center in the shape of triangles.
T: Can you explain what you mean by triangles?
S. The cubes look like the shape of a triangle.
T: Let's be sure everyone understands. Can you show us one of the triangles on the model you built of the fourth tower?
S. Sure. When you look at one of the arms coming out from the center [pulls the cubes away from the rest of the model], you have a piece with three cubes on the bottom, two on the middle level, and one on the top level. It looks like a triangle.
T: Okay, I see. Why are the triangles important?
S. Because if I can figure out how many cubes are in the triangles for each tower, I can add that number to the center tower and figure out how many cubes total. [The exchange continues as the student continues explaining.]

Level 0 The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher's questions.	Level 1 The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.
Level 2 The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another's work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.	Level 3 The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.