Haley Padgett

Grade: Elementary - Multiplicative Reasoning

Materials: Cubes, assignment on page 6

2. The Concept & Standards.

a. This lesson will help students understand multiplicative reasoning. The lesson is designed to provide students with the opportunity to learn about intensive and extensive quantities. By providing them with cubes, students will begin to distinguish the intensive quantity, the numbers cubes per tower, and the extensive quantity, the number of towers and total number of blocks. Throughout the lesson students will move their reasoning from more concrete examples to abstract thinking.

Indiana Standard:

3.AT.4: Interpret a multiplication equation as equal groups. Represent verbal statements of equal groups as multiplication equations.

Mathematical Process Standards:

PS.1: Make sense of problems and persevere in solving them.

PS.8: Look for and express regularity in repeated reasoning.

b. Students begin exploring multiplication and multiplicative reasoning with the use of grouping and repeated equal groups. As Wright mentions, students should be given the chance to build this skill by being given a set number of groups and the number within each group (2006). Within our lesson, Please Go Bring for Me, we have many opportunities for students to build their skill in repeated equal group counting by providing many concrete examples of towers and cubes per tower. Throughout the lesson, we hope to transition the students learning from physically working with the cubes to a more abstract thinking. With the use of the cubes, students can begin to think of abstract composite units (Wright, 2006). For example, when we ask how many cubes will there be in all if there are 5 towers with 3 cubes in each tower. The students will move from having to build each tower individually and counting each cube to thinking of 3 as an abstract composite unit and count 3 five times. With the use of cubes and keeping track on fingers, we want the students to understand multiplicative double counting. Throughout the exploration section of the lesson, we want the student to problem solve how they could keep track of the number of cubes in each tower and keep track of how many towers they have counted without the physical cubes in front of them. Many students will use their fingers to count the number of cubes and the number of towers, but this may become confusing or the student may run out of fingers. Through prompting, the student should transition to using skip counting and their own fingers or another way of keeping track of how many towers they

have counted (the teacher's fingers or tally marks). As students begin to understand unitizing and multiplicative double counting, they will become more comfortable and understanding of how multiplication really works.

Reference: Wright, R.J. (2006). *Teaching number in the classroom with 4-8 year olds.* London: Paul Chapman Publishing.

3. Outline of the Lessons.

a. Launch.

Begin by by telling the student that we are going to play a game called Please Go Bring for Me. We will bring out the cubes and allow the students to play with them for five minutes. Then we will explain the game by telling the student that we are going to make towers out of the cubes. Explain that we will ask them to imagine how many cubes there will be in all. WHen we ask them a question and they build a tower we will ask them three questions:how many towers do we have? How many cubes do we have per tower? How many cubes do we have in all? Explain to the student tha if they do not need to build every tower to find the answer then that's okay.

b. Explore, Rationale, Scaling.

i. Provide and describe the exploration (problem/task) and instructional goal(s):

Please Go Bring For Me: Students will move from physically making cube models to represent the two numbers in a multiplication equation to a more abstract understanding of multiplication. By making models, the students will begin to think about the groups and how many are in each group. This will lead them to grow their ability of unitizing and multiplicative double counting.

Goals: Unitizing, Multiplicative Double Counting

"Imagine I have 5 towers with 4 cubes in each tower. How many cubes do I have in all?"

"If you don't know, how about you go and make one tower with 4 cubes in it. (After the student has finished making one tower) Ok, now imagine I have 5 of these towers, how many cubes would I have in all?" (Continue until the student can compute on their own. They may or may not need to make all the towers). Ask the student how he came up with the answer.

Continue with different problems transition student from having to build every tower to only building one tower to not having to use the towers at all. Allow the student to use their fingers, your fingers, or tally marks to keep track of double counting.

Questions:

6 towers with 4 cubes in each

3 towers with 7 cubes in each

4 towers with 8 cubes in each

ii. Rationale: Yea

The goal of the lesson is for the students to understand unitizing and multiplicative double counting. Students should already understand one to one correspondence, counting on, and basic addition. By presenting them with the activity of building towers, we are showing them how to create units. Throughout the activity, we want them to not have to build every tower to answer the problem. On their own or with prompting, the students will begin to use our fingers or only use one tower to count and their finger to keep track of how many times they've counted to begin understanding multiplicative double counting.

The students will physically make cube models that represent two numbers in a multiplication equation. While the students are exploring the teacher will be providing prompts. When it seems appropriate, the teacher will prompt the student to not make every tower. The teacher may offer their fingers to represent how many towers there are.

Continue with different problems transition student from having to build every tower to only building one tower to not having to use the towers at all. Allow the student to use their fingers, your fingers, or tally marks to keep track of double counting.

iv. Student Strategies:

One student may not be able to figuratively think about multiplicative double counting. When asked, "Imagine I ask you to go bring me 6 towers with 4 cubes in each," he may not be able to equate how many cubes in all. When prompted, he will go make one tower of 4 cubes. Still, when asked how many would be in 6 towers, he can not figuratively imagine the rest of the towers. When prompted, he would build another tower of 4. When asked how many are in all, he could count each individual cube. For this student, he may have to build all 6 towers and count each individual cube to know how many there are in all.

Another student may be able to double count when given one tower. For example, when asked "Imagine I ask you to go bring me 6 towers with 4 cubes in each," he cannot equate the problem in his head. When prompted, he will go build 1 tower of 4 cubes. Then when asked how many cubes would be in 6 towers, the student may be able to answer the problem by keeping track on his hand how many towers, and counting the individual cubes in the one tower. This proves his understanding of double counting.

v. Scale up/Scale down

- Sarah: Scale up- My student is working on flashcards for multiplication. After going through the activity with him, I want to help draw the connection between the flash card/equations and the game we played. We will bring out a flash card and first go through with the first number being the number of towers and the second number being the number of cubes per tower. I want to work with him to draw that connection with my help a few times. Then I will switch to the first number being the number of cubes and the second number being the number of towers. This helps show the commutative property of multiplication. Also, this will help him understand his math facts and multiplication rather than just memorizing facts and numbers.

- Haley: Scale Down- My student is beginning to work on multiplicative thinking so to scale down this lesson I would focus on smaller numbers (numbers 1-5). For example, i will have the student build 2 towers with 5 cubes per tower. As the lesson continues I can ask the student to imagine how many cubes there would be if I added another tower with the same amount of cubes. As I am asking the questions it may be a good idea to write down the written equation so that he can make the connection between the towers and the multiplication equation. As we continue through the semester the student can begin working with larger numbers.

- Kelsey: Scale up: My student needs to work on learning the facts of 3, 4, 6, and 7. Therefore, if he finds the tasks less challenging, I could make the number of cubes and the number of towers larger such as having 6 towers with 12 cubes in each tower. I could use higher numbers and go up to 12's because it will provide more of a challenge for the student. I can also transition the student into not using the cubes to provide him more of a challenge to see if he is able to solve the different problems without the cubes. If he is able to master this activity, I may transition his thinking from how addition can be turned into multiplication. I could provide equation sentences like ___ + ___ + ___ + ___ = ____and ___ x ___ = ____ to see if he is able to understand how repeated addition can turn into a multiplication problem.

- Scale down: I could focus on smaller numbers such as having 3 towers with 4 cubes in each tower. I could use numbers smaller than 5 and below because it will allow the student to get a better understanding of the process and be able to work up into higher numbers. My student understands better when shown an example, so I could also guide him and model an example for him to be able to see the steps that I am taking and ask him questions as I am doing the problem. For an example, if I made 3 towers with 4

cubes in all, I may ask the student: “Imagine if I added one more tower with 4 cubes, how many cubes do I now have altogether?” to see if he can begin to think abstractly. My student also understands more when drawing a picture of the problem as well, so I may have him draw a picture and write an addition problem to give him a better understanding of what he is doing with the cubes and towers. Drawing the picture of the cubes and the towers gives him a better understanding of the process of unitizing and multiplicate double counting. I could have the student make all of the towers of the cubes at first and then slowly go from making less towers to no towers at all. I could have the student write out the problems that he is doing and then cover them up with a piece of paper and the student the questions again.

c. Summarize.

During this lesson, our students will be working on different types of work samples that requires them to use their critical thinking skills to discover how many cubes they have altogether when given a set amount of towers with equal number of cubes in each tower. To summarize, we can bring our students back together to talk about the ideas that they have learned while playing the game. Questions that we could ask to help the students grasp the concepts are, “How did you find the number of cubes in all when using the cubes?” “How did you find the number of cubes in all without using the cubes?” “Is there something you did every time to help you find the total number of cubes?” “How did you count the number of cubes in each tower?” “Did you group them together when counting?” “If so, what number did you count by?” “How did your addition problems change into multiplication problems?” “Is there something that you struggled with?”. This can help us see how the students received their answers and to see if they had to count each individual cube in each tower. We may see if the students grouped the items together to see if they gained the idea of unitizing and how each group is a unit. This may help us to see if the students gained the concept of double counting to see how they found the answer if they did not use the cubes and to see if they counted with their fingers or used a different strategy. We may also see if the students gained the concept of how repeated addition changed into a multiplication problem. By asking them all these questions and making them re-explain their thinking, it will reassure us of what they learned and give them an opportunity to express their knowledge on the concept.

4. Evidence of Student Understanding/Reasoning.

We will take evidence from the student's performance during the explore to figure out what the he has learned during the lesson. By observing the different methods the student uses to come up with answers we can figure out how the student is thinking and how well they are understanding the lesson. For example, if the student doesn't need to physically build the towers to come up with the correct answer then we can assume that he has a pretty good understanding of multiplication. We can assess the student on how well he is able to answer the questions: How many towers do we have?, How many cubes per tower?, and How many cubes

in all? We can use the assignment on the next page as our work sample and to gain what the student has learned and understandings. The assignment goes through multiple stages of reasoning and can show us what the student understand and still struggles with. The first problem makes sure that the student can accurately draw a representation of the groupings. Also, below the drawing it lays out their understanding that the number of cubes in each tower will be added. In the first problem, the number of towers (or how many times it is added) is provided in the blanks. However, in the second problem, they must show that they understand they are adding 8 cubes 4 times on their own as no blanks are provided. The second problem also asks the students to create a multiplication equation. This shows the teachers whether the students grasped the concept that repeated addition can be transitioned to multiplication. Finally, the last problem shows the teachers whether the students can produce the tower/cube drawings without explicitly being told how many towers and how many cubes. Based off their picture, we can gauge their learning and understanding of the concept. Also, by asking for a second way of drawing the equation, we can see if they have an understanding of the commutative property of multiplication. Even though it is not a goal for this lesson, seeing that understanding would be beneficial for their future learning.

5. Materials.

- Cubes
- Assignment on next page

6. Assignment.

1. Imagine I have 6 towers with 6 cubes in each tower. Draw a picture to represent each tower and the cubes in each tower.

   What is an addition equation that would represent your drawing?

   ___ + ___ + ____ + ____ + ____ + ____ =

2. Imagine I have 4 towers with 8 cubes in each tower. Draw a picture to represent each tower and the cubes in each tower.

What is an addition equation that would represent your drawing?

What is a multiplication equation that would represent your drawing?

____ x ____ =

3. Draw a picture for this multiplication equation.

3 x 5

Can you draw another way of representing 3 x 5?

7. Test problems:

1 - Imagine I have 6 towers with 4 cubes in each tower. How many cubes would I have in all? Draw a picture to show your answer.

2 - For the equation 5 x 6, if there are 5 towers, how many cubes are in each tower?

3 - Imagine I have 3 towers with 7 cubes in each tower. How many cubes would I have in all? Solve the problem without drawing a picture.